Optimization the Spherical-Statistical Optical Model parameters for fast neutron scattered from ^{90}Zr and ^{138}Ba Nuclei

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ARTICLE INFO	ABSTRACT
	The spherical-statistical optical model (SSOM) with Hauser-Feshbach coupled channel model (HFCCM) has been employed in optimizing the optical parameters for the scattering of fast neutrons scattered from ^{90}Zr and ^{138}Ba nuclei. Empirical formulae for optical parameters have been predicted for nuclei range $40 \leq A \leq 140$ and neutron energy ranges 1-20 MeV through analyses enormous sets of experimental data for scattering cross-sections with minimum accuracy. The calculated angular distributions for elastic and inelastic 8 MeV neutrons from ^{90}Zr and ^{138}Ba nuclei have been compared with standard experimental and evaluated data. Also, the real and imaginary volume integral parameters found a nuclear symmetry dependence and neutron energy with minimized the F-ratio test for the medium range of nuclei.
	Keywords: Spherical-Statistical Optical Model, fast neutrons, elastic and inelastic scattering cross-sections, ^{90}Zr nuclei, ^{138}Ba nuclei, Volume integral, optical parameters.

INTRODUCTION

The SSOM has been used extensively in the analysis, elastic scattering cross-section data for a wide range of projectile energies and nucleus target masses (Perey, 1963; Hodgson, 1971; Perey & Perey, 1976; Greenlees etel, 1968; Smith etel, 1987). The

investigation concentrated on the optimization the optical model with complex potential and adjustable parameters that consist with the behaviour of the experimental data, and also, diagnosed more information about the energy and the isospin dependence of the optical model potentials. Nevertheless, these parameters found by different authors are differed somewhat and make the interpolation between different energies and nuclei are uncertain. In addition the utilitarian value of such parameters, we believe that useful information about nuclear forces and structure can be obtained from the systematic analysis of neutron elastic scattering. For the purpose of phenomenological analysis the central term in the optical potential is taken to have real and imaginary potentials with their radius and diffuseness parameters V(r)=Uf(r) + i W g(r) , where the U and W are real and imaginary potential depths, and f(r) is the Saxon-Woods form factor, and g(r) is either Saxon-Woods (Volume Absorption) or derivative Saxon-Woods (Surface Absorption). This model was first proposed by (Serber & Phys, 1974) and used by (Fernbach etel, 1949) to calculate the scattering and absorption of 90 MeV neutrons by a range of nuclei. An extensive analysis work of experimental data by (Perey & Buck, 1962) indicated that the differences in optical parameters found by various investigators were mainly explained by the experimental uncertainties and differences in techniques of the optical model analysis used.

Also, they found that it was possible to find a single optical potential that gives cross-sections in excellent agreement with the data for medium and heavy nuclei for incident neutron energies from (1-25) MeV. From the work of (Perey & Buck, 1962), it seems the non-local optical potential give better results than those obtained with local potentials. While (Wilmore & Hodgson, 1964) viewed numerical complexity in number of adjustable parameters and calculated an equivalent local potential that gives good agreement with experimental data of cross-section for medium and heavy nuclei in the neutron energy range 1-15 MeV. A large amount of accurate data from proton and neutron elastic scattering, for energies less than 50 MeV, with medium to heavy nuclei, has been investigated by (Beccheti & Greenlees, 1969), which has allowed and improved set of nucleon-nucleus optical model parameters to be determined. With SSOM, (Hodgson etel, 1991) presented a good comparison concerns Hauser-Feshbach calculations with and without the width fluctuation correction. They calculated shape and compound elastic and inelastic scattering cross-sections for neutron of energy 0.2-2 MeV from ^{60}Co target nuclei. The properties of SSOM and coupled channel model of Hauser-Feshbach in the region of the doubly closed shell at A-208 and the neighbouring ^{209}Bi are examined by (Smith, 2007) with a particular attention to the energy dependence of imaginary potential that gives a good description of the observed neutron cross-section.

THEORY

In SSOM the present considerations are extensions of the conventional spherical surface absorption neutron optical model. The optical potentials consist of the Woods-Saxon volume and surface derivative forms which can be described as:-

$$V_{op}(r) = U_R(r) + i\, U_I(r) + U_{so}(r) \qquad (1)$$

where $U_R(r)$: is the real central potential and its form is taken to be a Woods-Saxon Well (Woods & Saxon, 1954; Bespalova etel, 2002), $U_I(r)$, is constitutes of the imaginary control potential which is responsible for the absorption, that occurred within the nuclear volume (is proportional to the Woods-Saxon Well) and near the nuclear surface (is proportional to the derivative of the Woods-Saxon Well), and $U_{so}(r)$ is the spin-orbit potential that has a Thomas form with the radial variation (Hodgson, 1971; Hodgson, 1984).

So the optical-model potential becomes:-

$$V_{op}(r) = -V_R\, f(x_\circ) + \left(\frac{h}{m_\pi c}\right)^2 V_{so}(\bar{\sigma}\bullet\bar{\ell})\frac{1}{r}\frac{df(x_{so})}{dr} - i\left[W_V f(x_{IV}) - 4W_D \frac{d}{dx_{ID}} f(x_{ID})\right] \quad (2)$$

The functions $\left[f(x_\circ), f\left(x_{so}\right), f\left(x_{IV}\right), f\left(x_{ID}\right)\right]$ are Woods-Saxon form factors with appropriable radius and diffusivity parameters in real, spin-orbit, volume and surface terms respectively.

In more detail the conventional shape-elastic scattering cross-section formulae at low and high incident neutron energies can be found in (Serber & Phys, 1974; Hodgson etel, 1991; Smith, 2007; Hodgson, 1984; Lawson & Smith, 1999).

The HFCCM provides a prescription for dividing the compound cross-section into the compound elastic scattering and other channels in which the nucleus is primarily left in an excited state and the neutron is emitted with reduced energy (Serber & Phys, 1974; Fernbach etel, 1949; Hodgson etel, 1991). The total compound elastic or inelastic scattering cross-section for emitting neutron with energy E_f leaving the nucleus in the final state can be calculated from a knowledge of the transmission coefficients W_i in all channels and the energies, and the angular momenta of the states of the residual nuclei becomes (Lawson & Smith, 1999):

$$\sigma_{HF}(E_f) = \frac{\pi\,\lambda}{2(2J_i+1)}\sum_{jl}\left\{W_{jL}(E)\,\delta^J_{jJ_i}\sum_{j'l'}W_{j'l'}(E_f)\,\delta^J_{j'J_f}\,/W_a\right\} \quad (3)$$

where the transmission coefficient of the neutron emission in the channel $a\,(E',j',l')$ is

$$W_a = \sum_{j''l''O}W_{j''l''}(E_O)\,\delta_{j''J_O}, \quad \delta_{jJ_O}=1\ if\quad |J_O-j| \le J \le (J_O+j)\ \text{or equal zero otherwise,}$$

E_O is the energy of the outgoing neutron in a channel O and J_i is the spin for the ground state.

The insensitivity of the shape and compound elastic scattering cross-section to variations in the depth (U) and radius (r_R), of the real part of the potential that maintain the constancy of $(U\,r_R^2)$ (Greenlees etel, 1968), suggests that it is useful to adopt the Volume integral of the real and imaginary potentials (VIRP and VIIP) as a relatively invariant function of the parameters, and to investigate its variation with energy and nuclear asymmetry parameter.

Therefore, the volume integral per nucleon defined as:-

$$J = \frac{1}{A}\int V(r)\,dr \quad (4)$$

For a potential of Woods-Saxon form, ie. $U = -V\left\{1+\exp\left[(r-R)/a\right]\right\}^{-1}$, $R = rA^{1/3}$ and $U = U_R + U_D + U_{so}$, the volume integral becomes:-

$$J = \frac{4\pi}{3}\frac{R^3 U}{A}\left\{1+\left(\frac{\pi a}{R}\right)^2 + \left[\left(\frac{a}{R}\right)^4\right]\right\} \quad (5)$$

SPECIFICATION OF THE CALCULATIONS AND ANALYSIS THE RESULTS

An extensive series of calculations of 1-20 MeV neutron elastic and inelastic scattering cross-section on different nuclei has been achieved using the SSOM with HFCCM. In this work nine optical parameters are considered (V_R, W_I, W_{so} and their radius and diffuseness) and compared with standard experimental data, EXFOR (EXFOR, 2014), and standard Evaluation data, ENDFVII (ENDF/B-VII-1, 2014). The optical parameters have been optimized through the minimization of chi-square per point, within the range 0.81-2.04 for the range of target nuclei $40 \leq m_T \leq 141$ *amu*, and compared the calculated results with experimental data of (ENDF/B-VII-1, 2014). The optimized optical parameters [$V_R, r_R, a_R, W_I(W_V, W_D), r_I, a_I, W_{SO}, r_{SO}, a_{SO}$] are formulated using the multiple regression analysis and indicated that the real and imaginary potentials are nuclear asymmetry, ξ, dependence, and neutron energy, while the other optical parameters are fixed as shown in table (1).These parameters differ from (Engelbrecht & Fiedeldey, 1967).

A comparison with the date of (EXFOR, 2014; ENDF/B-VII-1, 2014) show a quite successful agreement in behaving the angular distribution in the energy 8 MeV for ^{90}Zr target nuclei, figure (1), and 14 MeV neutrons on ^{138} a target nucleus, figure (2). In table (1) a summary of the predicted optical parameters that weighted with experimental data of (EXFOR, 2014) and (ENDF/B-VII-1, 2014) through F-ratio and square residuals. The angular distributions of inelastic scattering cross-section of 8 MeV and 14 MeV neutrons from Zr and Ba nucleus, have been investigated from a continuum excited states using HFCCM. Figure (3) shows a good agreement for the neutron scattering cross-section from 1.751 MeV and 2.186 MeV excited states of ^{90}Zr nuclei compared with experimental results of (EXFOR, 2014). Since no experimental data available for a neutron inelastic scattering cross-section of the continuum states of ^{138}Ba nuclei, therefore the present work explores the angular distributions from five excited states (1.453 MeV, 2.09MeV, 2.189MeV, 2.203MeV, 2.217MeV), as shown in figure (2). Similarly, the volume integral, equation (5), has been formulated for a wide set of experimental data via optical potential $(V_{OP}(r))$ in case of real and imaginary potentials. Figure (4) shows the behaviour of the empirical formula of the volume integral against a neutron energy range 1-20 MeV for ^{90}Zr nuclei, where the comparison done with the calculated ABAREX code (Lawson & Smith, 1999), and shows an acceptable agreement in the case of the real potential volume integral.

In general, the theoretical calculations indicated that the probability of the Shape elastic scattering in the forward directions is larger than that probability of the compound elastic and become less difference in the backward directions, due to the neutron energy status, that caused the incident particle may be absorbed to form the compound nucleus, which subsequently re-emits a similar particle back into the entrance channel. Since, experimentally the probability of the compound elastic is indistinguishable from the shape elastic scattering, therefore, the theory of the optical model can counted as part of the compound nucleus cross-section.

CONCLUSION

The SSOM with HFCCM have been used to analyaze the optical model parameters (OMPs) for range of target nuclei $40 \leq m_T \leq 141$ *amu* and at neutron energy range 1-20 MeV. The optimization of the OMPs [$V_R, r_R, a_R, W_I(W_V, W_D), r_I, a_I, W_{SO}, r_{SO}, a_{SO}$, *VIRP*, *VIIP*] have been obtained through the minimization of chi-square per point, within the range

0.81-2.04, by comparing the calculated data with the experimental data of (ENDF/B-VII-1, 2014). It was concluded that the OMPs are nuclear asymmetry, ξ , dependence, and neutron energy, while the other optical parameters are fixed as shown in table (1). These parameters differ from (Engelbrecht & Fiedeldey, 1967).

Also, from the analysis of the OMPs, it concluded that the imaginary potential W_I value, which has the Saxon-Woods derivative forms, is a nuclear asymmetry and neutron energy dependence rather than neutron energy dependence mentioned by (Hodgson, 1971). This conclusions also withdrow for the dependence of VIRP and VIIP parameters on neutron energy and nuclear asymmetery.

The calculated angular distributions for elastic and inelastic scattering cross-sections for ^{90}Zrand ^{138}Ba target nuclei, at 8 and 14 MeV respectively, show good consistency with theoretical, experimental and evaluated data of (Lawson & Smith, 1999; EXFOR, 2014; ENDF/B-VII-1, 2014), while no experimental data found for a neutron inelastic scattering cross-section of the continuum states (1.453 MeV, 2.09MeV, 2.189MeV, 2.203MeV, 2.217MeV) of ^{138}Ba nuclei.

Table (1): The neutron energy and nuclear asymmetry dependence for empirical form parameters calculated in terms of SOM, for target masses $40 \leq A \leq 141$ amu .

| Parameter	Empirical formula		F-Ratio	R^2
Real potential	$V_R = 51.523 - (0.32 \pm 0.133)E_n - (42.581 \pm 22.335)\xi + (0.0581 \pm 0.032)A$	(MeV)	1.90	2.0
	$r_R = 1.17$	(fm)	-----	-----
	$a_R = 0.75$	(fm)	-----	-----
Imaginary Potential	$W_I = 6.672 + (0.173 \pm 0.069)E_n + (33.209 \pm 11.518)\xi - (0.059 \pm 0.016)A$	(MeV)	5.94	1.0
	$r_I = 1.26$	(fm)	-----	------
	$a_I = 0.58$	(fm)	-----	------
Spin-Orbit Potential	$V_{SO} = 6.2$	(MeV)	-----	-----
	$r_I = 1.1$	(fm)	-----	-----
	$a_R = 0.75$	(fm)	-----	-----
VIRP	$J_R = 476.107 - (2.125 \pm 0.708)E_n - (132.004 \pm 175.065)\xi - (0.362 \pm 0.284)A$	$(MeV.fm)^3$	13.99	20.0
VIIP	$J_I = 100.078 + (1.963 \pm 0.463)E_n + (303.853 \pm 101.63)\xi - (0.855 \pm 0.162)A$	$(MeV.fm)^3$	27.78	12.0
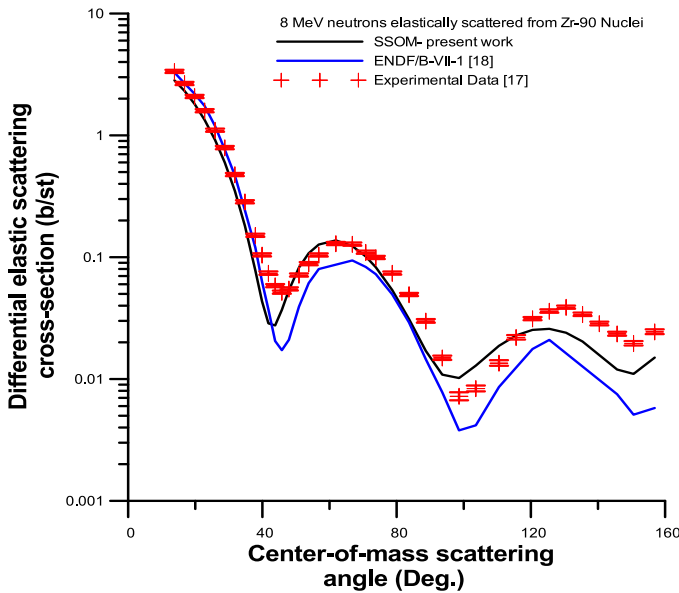
Figure (1): The angular distribution of differential elastic scattering cross-section for 8 MeV neutrons scattered from ⁹⁰Zr nuclei compared with experimental data, (EXFOR, 2014) and **ENDF/BV-II-1**(ENDF/B-VII-1, 2014).
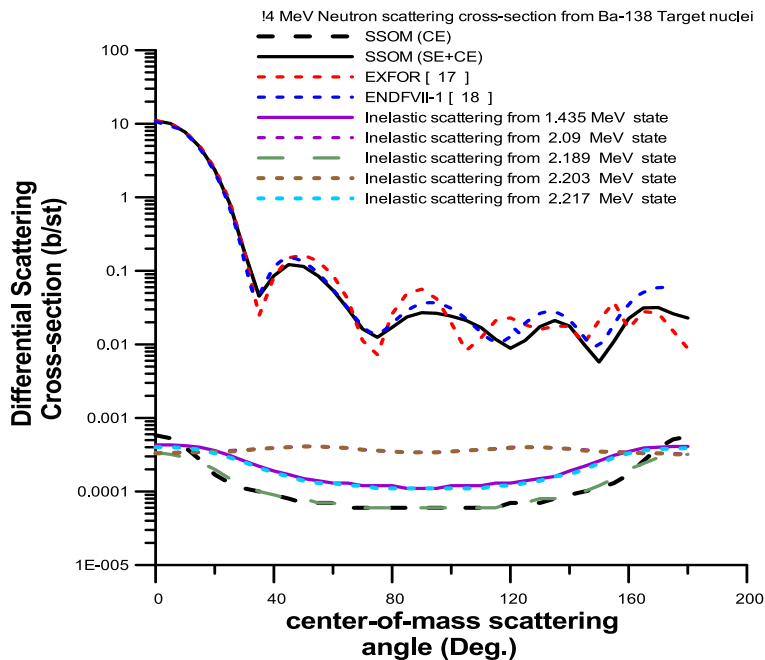

Figure (2): The differential elastic and inelastic scattering cross section of the 14 MeV neutron form ¹³⁸Ba nuclei compared with experimental data, **EXFOR**, (EXFOR, 2014) and ENDF/B-VII-1(ENDF/B-VII-1, 2014).The optical parameters [V_R,r_R,a_R,W_I(W_V,W_D),r_I,a_I,W_{SO},r_{SO},a_{SO}] are 50.65 MeV, 1.17fm, 0 .75fm, 4 8.49 MeV, 1.26fm, 0.58fm, 6.2MeV, 1.1fm and 0.75fm respectively.
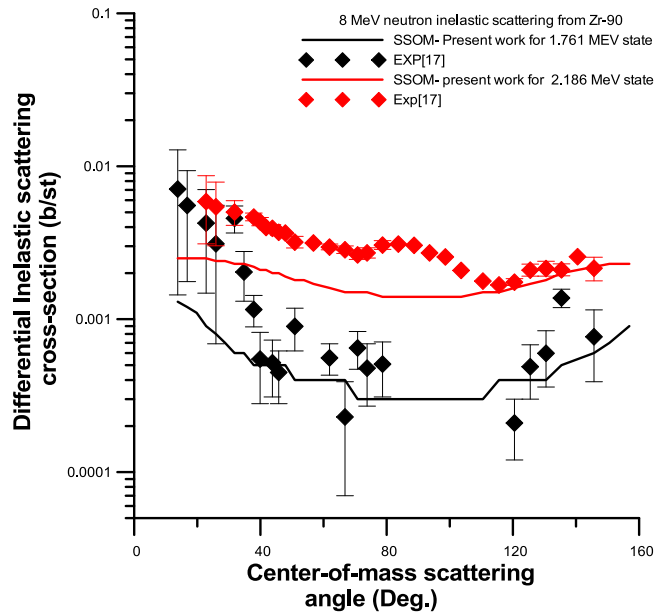
Figure (3): The angular distributions of inelastic scattering cross-section of 8 MeV neutrons from 1.761 MeV and 2.186 MeV excited states of ^{90}Zr nuclei compared with experimental data of (EXFOR, 2014).
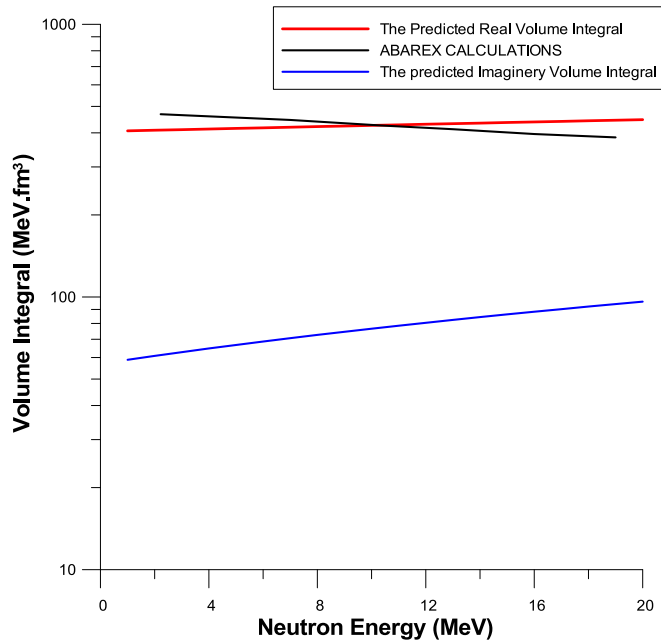

Figure (4): The predicted real and imaginary volume integral as a function of neutron energy for ^{90}Zr nuclei compared with **ABAREX** code calculations (Lawson & Smith, 1999) for the real volume Integral.

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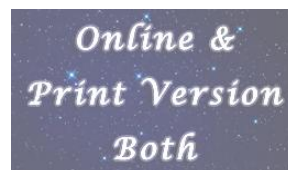
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